

## Simulation of vertical dynamic interaction between railway vehicle and slab track

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Department of Mechanics and Maritime Sciences  
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THESIS FOR THE DEGREE OF LICENTIATE IN  
SOLID AND STRUCTURAL MECHANICS

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Cover:

Sketch of one of the implemented vehicle–track interaction models. The vehicle is modelled as a multibody system, and the track is modelled using finite elements. The track model contains three layers of beams: rail, discrete panels of concrete slab and continuous concrete roadbed. The concrete roadbed is supported by a Winkler foundation.

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## ABSTRACT

The usage of slab track for high-speed railway lines has increased in recent decades. In Sweden, the building of new railway lines for higher speed and the selection of track design for such lines are currently being debated. Slab track has, so far, only been applied in small scale in Sweden, which implies that the knowledge and experience of such track are limited. This thesis aims to improve the understanding of the dynamic interaction between railway vehicle and slab track.

The vertical dynamic vehicle-track interaction is simulated in the time domain using an extended state-space vector approach. By using a complex-valued modal superposition technique for the considered linear, time-invariant and two-dimensional track models, the computational cost of solving the associated initial-value problem is reduced. Two generic slab track models, including one or two layers of concrete slabs, are presented. The upper layer of the two-layer slab track model is described by decoupled beams of finite length, while the lower layer is a continuous beam. From the solution of the initial-value problem, wheel-rail contact forces, bending moments in the concrete panel and load distributions on the supporting foundation are evaluated. The presented models are applied to calculate the influences of track design parameters on various track responses. Furthermore, the influences of longitudinal track stiffness gradients and rail imperfections causing periodic and transient excitations are analysed.

Transition zones between the one-layer slab track model and a ballasted track model are analysed. By considering a multi-objective optimisation problem solved by a genetic algorithm, the maximum dynamic loads on the track structure are minimised with respect to the selected design variables. From the solution of the optimisation problem, a non-dominated front of the objective functions is obtained illustrating potential for a significant reduction of the dynamic loads. Since the transition zones are optimised neglecting the influence of wheel and rail irregularities, a methodology is proposed to assess the robustness of the optimal design by evaluating its performance when periodic rail irregularities with different combinations of wavelength and phase, relative to the position of the transition, are applied in the model.

Keywords: Slab track, dynamic vehicle-track interaction, transition zones, genetic algorithm, modelling, ballastless track.



## PREFACE

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Gothenburg, May 2018

Emil Aggestam





# THESIS

This thesis consists of an extended summary and the following appended papers:

## Paper A

E. Aggestam, J. C. O. Nielsen and R. Bolmsvik. Simulation of vertical dynamic vehicle–track interaction using a two-dimensional slab track model. *Vehicle System Dynamics*. (2018).

## Paper B

E. Aggestam and J. C. O. Nielsen. Dynamic interaction between vehicle and slab track – Influence of track design parameters. *Proceedings of the 25th International Symposium on Dynamics of Vehicles on Roads and Tracks (IAVSD 2017)*, Rockhampton, Australia. (2017), 699–704.

## Paper C

E. Aggestam and J. C. O. Nielsen. Multi-objective optimisation of transition zones between slab track and ballasted track using a genetic algorithm. *Submitted for international publication*. (2018).

The appended papers were prepared in collaboration with the co-authors. The author of this thesis was responsible for the major progress of the work including taking part in the planning of the papers, developing the theories and the numerical implementation, performing the numerical simulations and writing the papers.



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# Part I

## Extended summary

### 1 Introduction

The construction of the railway infrastructure started in the 19th century, and railways are today a part of the everyday life for many people. During the years when people have used the railway transport mode, it has been developed with respect to comfort, safety, reduced degradation, etc. To make transport of freight and people by train a competitive transport mode, the railway industry strives towards using increased axle loads and higher speeds of the trains. However, increasing the axle load and speed leads to a higher deterioration rate of both the track and the vehicle due to increased dynamic loads. Today, the so-called ballasted track<sup>1</sup> is the most common railway track design. A disadvantage of this type of track is that the track geometry may have a high degradation rate leading to frequent and expensive maintenance [1, 2]. To mitigate the deterioration of the track, alternative track designs have been developed. One of the most promising designs, which is already being used for high-speed railway applications in several countries, is the so-called slab track.

In a slab track, which is also called a ballastless track, the sleepers are replaced or combined with slabs, which are large (continuous or discrete) concrete plates. In doing so, a stiffer track structure is achieved with a longer life. Other advantages of slab track compared to ballasted track are reduced needs for maintenance, reduced height, larger accessibility and eliminated problems associated with degradation of ballast [2]. There are, however, also disadvantages with slab tracks, such as higher construction cost and lower vibration/noise absorption.

In this thesis, the aim is to improve the understanding of the vertical dynamic vehicle-track interaction when slab tracks are used. In **Paper A**, a methodology to simulate the vertical dynamic vehicle-track interaction is presented, and the performance of the procedure is exemplified by three demonstration examples. In **Paper B**, a parametric study of the methodology is performed to investigate the influences of track design parameters and foundation stiffness gradients on various track responses. Finally, **Paper C** analyses transition zones between slab track and ballasted track. A multi-objective optimisation methodology is applied to minimise the dynamic loads in the transition zone.

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<sup>1</sup>In a ballasted track, the rails are mounted on sleepers (beams usually made of wood or concrete) supported by ballast.

## 2 Design of slab track systems

In this chapter, both high-speed railway lines in general and different slab track systems are described and discussed. In particular, Section 2.1 gives a brief review of the history of the development of railway lines, current designs of high-speed railway lines and what challenges the railway community need to handle in the future to make the transport of freight and people by train a competitive option compared to other transport modes. In Section 2.2, an overview of different types of slab track systems is presented. The emphasis in this section lies in the description of slab track systems using prefabricated concrete slabs since this type of system is modelled and simulated in the appended papers.

### 2.1 High-speed railway lines

The construction of railways started in the early 19th century during the industrial revolution [3]. One of the most famous projects is the so-called “Rocket” locomotive that travelled at 50 km/h and was designed by George Stephenson in 1829. Already in the beginning of the 20th century, top speeds over 200 km/h were recorded. However, in the 1930s, the highest average speed between two cities was still only 130 km/h. In 1964, the first high-speed line, called Shinkansen, started to operate in Japan at vehicle speed 210 km/h (later increased) over a distance of 515 km<sup>2</sup>. Initially, only ballasted track was used, but in 1972 the first sections of slab track were installed, and by 1993 over 1 000 km of slab track had been implemented [4]. In 1981, the first high-speed line in Europe started operating in France with a vehicle speed of 260 km/h and it was built using ballasted track [3]. From the 1980s and onwards, several countries have built high-speed lines. In particular, China has since 2008 implemented approximately 20 000 km of high-speed lines (mainly built using slab track) and carries today more than half of the high-speed railway traffic in the world. Today, some countries, e.g. Germany, Japan and China, prefer to use slab track, while other countries, e.g. France and Spain, prefer ballasted track. The high-speed railway networks in Europe and Asia (including systems under construction) are illustrated in Figures 2.1 and 2.2, respectively.

High-speed railway lines are complex systems including infrastructure, stations, rolling stock, operations, maintenance strategy, financing, marketing and management [3]. On top of that, railway lines may be different between different countries in terms of commercial approach, operation criteria and cost management. However, high-speed railways is still a competitive transport mode since it offers high capacity and is environmentally sustainable. When high-speed lines are used, specialised trains and railway lines are required. In these trains, the reliability, aerodynamics and safety are increased, whereas for the tracks, the track quality, catenary, and track layout are improved.

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<sup>2</sup>Today, the criterion for high-speed railway lines is operating speeds of at least 250 km/h [3]. Even though the Shinkansen originally only operated at 210 km/h, it is commonly referred to as the first high-speed line.

During recent decades, also ballasted tracks (and related rolling stock) have been improved significantly. In parallel, new solutions have been developed that need to be considered when new high-speed lines are built. The UIC (International Union of Railways) states that one key aspect in order for the railways to be further improved and stay as a competitive transport mode in the future, is that the slab track technology needs to be further assessed [3]. Today, it is a difficult task to determine whether slab track or ballasted track is the most beneficial track design for high-speed lines, see Chapter 3. In the decision process, several different parameters need to be taken into account, where the most important parameters are operational conditions (traffic characteristics), technical infrastructure features (viaducts, tunnels, local geotechnical features) and environmental conditions (noise, vibration, CO<sub>2</sub> footprint).



Figure 2.1: High-speed railway lines in Europe (2015). From UIC [3].

## 2.2 Overview of slab track systems

Although the use of slab track structures for high-speed applications has grown during recent decades, the technology is in several aspects similar to the system that was installed in Japan 1972 [4]. However, during these years when the technology has been optimised, several types of slab track systems have been developed. In this thesis, currently existing



Figure 2.2: High-speed railway lines in Asia (2015). From UIC [3].

slab track systems are divided into direct rail support systems and continuous rail support systems, as previously done by Esvelde [1].

### 2.2.1 Discrete rail support systems

In discrete rail support systems, which are the systems that dominate high-speed slab track lines, the rail is supported at discrete, equidistant locations. Discrete rail support systems can be further divided into systems with or without sleepers [1].

When sleepers are used, they are either embedded in the slab or placed on top of the slab. One of the most famous designs with sleepers embedded in the slab is the so-called Rheda system [1]. The first application was developed in the 1970s, and in 2000 an upgraded version of Rheda, called Rheda 2000, was installed. In Rheda 2000, the sleepers consist of a concrete filigree twin-block design, which ensures a precise location of the rails. The sleepers are embedded in a concrete slab, which is supported by a hydraulically bound layer (HBL). Today, Rheda 2000 has successfully been installed in Germany, the Netherlands, Taiwan and Korea [2]. Rheda 2000 and prefabricated slab track systems (that will be discussed below) are commonly called compact systems, since the height from slab track base to rail head is less than 0.5 m.

When sleepers are not used, prefabricated concrete slabs or monolithic slabs are used. Continuous monolithic slab track structures, which are particularly well suited on civil structures such as bridges, are not built in any larger scale for high-speed railway lines. In this thesis, the modelling of prefabricated concrete slab track structures is considered. This type of slab track is chosen since it is the most spread and common type of slab



track in the world. When comparing prefabricated slab track systems to other slab track designs, the main advantages are that prefabricated systems are maintenance friendly, have a high quality due to a high level of automation and a short construction time. Today, a lot of different prefabricated slab track structures are available on the market, where the most well-known designs are the Shinkansen, Feste Fahrbahn Bögl (FFB), ÖBB-Porr (also called Slab Track Austria, STA) and the China Railway Track System (CTRS) series. Two examples of the STA system are shown in Figures 2.3 and 2.4. Figure 2.3 illustrates a system during construction, whereas Figure 2.4 illustrates a system ready for traffic.

Below, a summary of the four most wide-spread prefabricated slab track systems is presented. The typical layers used in prefabricated slab track systems are shown in Figure 2.5, and contain (from top to bottom); rails, prefabricated concrete panels, (elastic slab mats), filling, concrete roadbed, (frost protection layer) and foundation/soil<sup>3</sup>. Note that both of the designs expressed in Figure 2.5 is modelled in this thesis, see **Papers A–C**.



Figure 2.3: The STA system during construction. Note that the asphalt used as a base layer for the track in the photo is normally not used by STA. From STA [5].

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<sup>3</sup>The layers within parentheses are not always installed depending on the geographical location and the design of the slab track.



Figure 2.4: Final STA system ready for traffic. From STA [5].

### Feste Fahrbahn Bögl

The different layers used in the Feste Fahrbahn Bögl (FFB) slab track are illustrated in Figure 2.5(a). In the longitudinal direction, the reinforcement of the prefabricated panels with length 6.45 m is extended and coupled to the reinforcement of the adjacent panels during construction [1]. By using such a design, the final installation of the individual panels can be seen as one continuous panel that rests on the filling (bituminous-concrete mortar) and the roadbed (plain concrete). If the track is installed in a cold climate, a frost protection layer (FPL) is placed underneath the roadbed. In the FFB slab track design, no bollards are used which implies that longitudinal and lateral movements between the different layers are only prevented by friction. The slab track design by FFB has mostly been installed in Germany and China [2].

### ÖBB-Porr

The slab track manufactured by ÖBB-Porr, sometimes also called Slab Track Austria (STA), has been a part of the railway infrastructure in Austria since the 1989 and is today mostly used in Austria, Germany and Qatar [2]. STA uses a prefabricated concrete panel of length 5.2 m with two rectangular “holes”. In these holes, self-compacting concrete (SCC) is poured during the construction (when the height of the panels has been fixed), which prevents movement in the longitudinal and lateral directions. An elastic layer is integrated on the bottom of the panel to make quick panel replacements possible and to

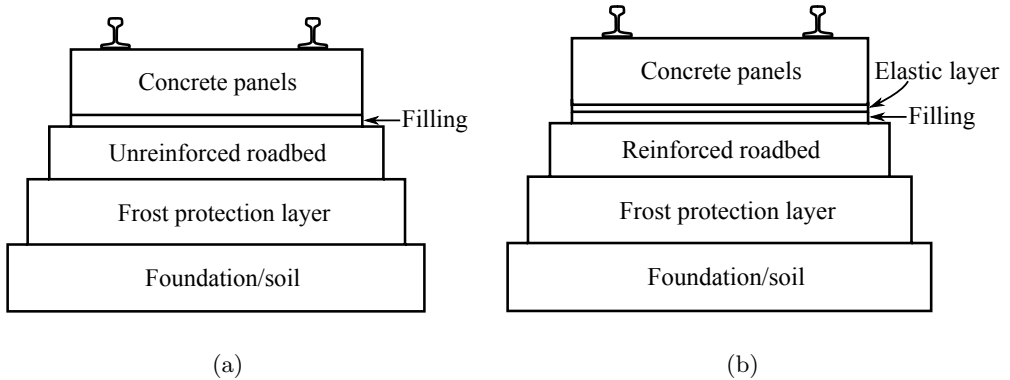


Figure 2.5: Cross-sections of two typical designs used for slab track systems. The track systems correspond to (a) German design with hydraulically bound layer and (b) Austrian design with load distribution slab. Note that also the Shinkansen slab track and the CTRS series have similar cross-sections.

reduce ground vibrations. Similar to the FFB system, the STA system rests on a concrete roadbed (which in the STA design is reinforced), and if the track is installed in a cold climate, an FPL is placed between the roadbed and the foundation. The different layers of the STA design are shown schematically in Figure 2.5(b).

## Shinkansen

The development of the Shinkansen slab track started in the 1960s, and the first high-speed railway line using this type of slab track (which was the first commercially used high-speed slab track in the world) started operating in the 1970s [1]. The Shinkansen slab track consists of several layers of different materials, similar to the layers sketched in Figure 2.5. Each prefabricated concrete panel of the Shinkansen slab track is approximately 5 m long and has a semi-circular cut (with a radius of 0.3 m) at the centre of both ends in the longitudinal direction. When the prefabricated slabs are put together, circles are formed, where bollards are placed in order to prevent movements in the lateral and longitudinal directions.

## China Railway Track System

The China Railway Track System (CTRS) series consists of three generations of slab track designs, which are usually denoted CTRS-I, CTRS-II and CTRS-III. In all these designs, the rail is discretely supported by a reinforced concrete panel [6]. The CTRS-I consists of discrete panels of 5 m and uses bollards to prevent motion in the lateral and longitudinal directions (similar to the Shinkansen design), whereas CTRS-II and CTRS-III are continuous with longitudinal joints (similar to the FFB design). In all the designs, the panels rest on a filling layer, a roadbed made of concrete (reinforced concrete for CTRS-I and plain concrete for CTRS-II and CTRS-III) and a subgrade made of broken stones.

For CTRS-I and CTRS-II, the filling is made of cement-asphalt (CA) mortar, whereas self-compacting concrete is used for the CTRS-III.

### **2.2.2 Continuous rail support systems**

In continuous rail support systems, the rail is continuously clamped or embedded in an elastomeric layer [1]. By using a continuous rail support system, the dynamic loads are reduced compared to direct rail support systems due to the absence of the periodic variation in track stiffness from the discrete rail seats. Further, the degradation of the rails is reduced, and the need for maintenance is lower compared with other types of slab tracks. One disadvantage of continuous rail support systems is that a settlement-free soil is required due to a minimal possibility of rail readjustment.

Continuous rail support systems can be divided into embedded rail system, also called embedded rail structure (ERS), and rail structures that are clamped and continuously supported [1]. In ERS, the rail is fixed by an elastic compound, typically cork or polyurethane, which surrounds the entire rail except for the rail head. During construction, the elastic compound is poured into the track using a groove. In the last 40 years, ERS has been built in several pilot projects in the Netherlands. Although the benefits of the ERS were verified in these projects, the usage of ERS for high-speed lines is still rather limited (probably due to the requirement of a settlement-free soil). In rail structures that are clamped and continuously supported, the rail is continuously supported without using any elastic compound. Examples of designs are the Cocon track and tracks using web-clamped rails. The usage of these designs is, however, limited and they have not been installed in long-distance high-speed railway lines.

### 3 Slab track versus ballasted track

When comparing slab track and ballasted track, an advantage when using slab track is that the track geometry deteriorates at a slower rate and less maintenance work is required. Hence, the availability of a slab track is higher, which increases the capacity if the track is fully utilised. One of the main reasons why the deterioration rate is higher for the ballasted track is that the track structure is relatively soft in the lateral and longitudinal directions due to the discrete supports by sleepers on ballast. In particular, this is a problem in curves since the lateral resistance provided by the ballast is limited. Further, when a ballasted track is exposed to traffic in the long-term, parts of the ballast may be displaced which leads to an uneven support of the sleepers [1]. Therefore, the support conditions of the sleepers vary significantly along the track, and the appearance of hanging sleepers is common. The uneven support by the ballast, combined with dynamic loads from vehicles, cause movements of the sleepers which induce ballast degradation and differential track settlement. Poor quality of the track geometry leads to increased dynamic loads, which further increases the differential settlement. Another problem associated with the degradation of ballast is that fine particles are created from the ballast due to wear and fracture, which may cause drainage problems.

When the maximum allowed speed on the railway line is increased, the requirements on the track design become harder to meet. In particular, if the speed is increased, the minimum curve radius has to be increased. Therefore, more tunnels and bridges are typically required for high-speed lines. Slab tracks are particularly well-suited to tunnels and bridges since no ballast is required. Hence, the railway track can be integrated with the roadbed on bridges and (due to the low height of the superstructure) the cross-section of tunnels can be smaller. Further, when slab tracks are used in tunnels, the accessibility of road vehicles is easily integrated, which is required in case of an emergency [1].

When comparing the costs of slab track and ballasted track, the general conclusion is that slab track is more expensive to build, but cheaper to maintain and has a longer life. Since the decision of whether building slab track or ballasted track may have tremendous economic effects, published results describing all the decisions made and including an assessment of the costs are limited [2, 7]. In a report from 2001, Ando and Sunaga [8] concluded that the maintenance cost of the Sanyo Shinkansen slab track line was 25% of the maintenance cost for a ballasted track, but the construction cost of the line was 30 – 50% higher. LCC (Life-Cycle Cost) analyses comparing ballasted track and slab track are, generally, not available since the input to the analysis varies with topography, climate, operational conditions, etc. [2]. Further, when LCC analyses are performed, there are a lot of uncertainties, where two of the most critical ones that need to be assessed before a track is built is the cost of building slab track and (if high speed is considered) the maintenance cost for ballasted track. Moreover, it shall be noted that ballasted track is a thoroughly tested track design that has been used for over 150 years [7]. Therefore, track engineers know what types of problems to expect and how to handle them.

In slab track design, the emphasis is typically put on the slab track itself, while the roadbed and foundation are designed using general platform design rules [2]. The require-

ments set on the roadbed and foundation are, however, crucial since differential settlement of the slab track must not occur. In the European standard EN 16432-2:2017 [9], the  $E_{v2}$ -value (which is a measure of the stiffness of the foundation) is required to be at least 120 MPa. For track on soft soils, a remedy is to use a so-called settlement free plate (SFP), which is a viaduct-like structure that the slab track is placed on [2]. SFPs have successfully been used in the Netherlands and China.

Slab track is generally a stiffer structure than ballasted track. The track stiffness at the rail level of slab tracks is typically reduced by using a soft rail pad. Using a soft rail pad is, however, non-beneficial from an acoustic point of view since rail vibrations are increased [10]. Further, ballast is a good absorber, whereas the slab (if not treated) is a noise reflector. These noise problems lead to an overall noise increase of about 3 dB for slab tracks. Actions have been made to reduce the higher noise associated with slab tracks, e.g. by installing absorbent panels, but this is still an area that needs to be further investigated.

According to the United Nations, a well functional and sustainable infrastructure is crucial for humans in the future [11]. One step in order to achieve this is a state-of-the-art railway transport mode. However, due to the high installation cost of new track, the technologies used must be thoroughly investigated. To decide whether slab track or ballasted track should be used on a given line is a difficult question for the infrastructure manager, but to increase the odds of making the right decision, all different aspects of building slab track and ballasted track have to be addressed.

## 4 Transition zones

In a transition zone, there is a structural discontinuity which leads to an abrupt variation in the track stiffness and increased dynamic loads [12]. Traditionally, the track stiffness is typically changed due to a transition between two different substructures, e.g. embankment to bridge or tunnel. With the increased usage of slab track, the number of transition zones where a change in the superstructure design causes the variation in track stiffness has grown. Independently of the type of transition, the abrupt variation in track stiffness yields increased dynamic loads which may lead to accumulated differential ballast/subgrade settlement and irregularities in track geometry (longitudinal level). The accumulated settlement and increased track irregularities magnify the dynamic loads further which creates a vicious circle. Hence, the track area around a transition is prone to deteriorate at an accelerating rate, and frequent maintenance work is required. In transition zones, the number of required maintenance actions may be three to eight times higher compared to a conventional track [13]. Further, performing track maintenance work is a costly process. As an example from 2015, 54% of the investment cost for infrastructure (5199 million Euros) in Spain went to railway and over half of this sum was spent on maintenance of track and infrastructure materials.

To decrease the required maintenance work associated with transitions, various types of transition zones are installed. In different countries, different approaches have been applied. For most of the approaches, the key idea is to reduce the dynamic loads by making the softer track stiffer and the stiffer track softer in the vicinity of the transition. In doing so, a gradual change in the vertical track stiffness can be achieved, and the degradation of the track is reduced. When considering transition zones between ballasted track and slab track, a smoothing of the vertical track stiffness is typically achieved by varying the stiffness of the rail pads, the sleeper length and/or the sleeper spacing close to the transition, adding auxiliary rails at the transition and/or improving the stiffness distribution of the foundation [14]. The modelling of transition zones is described in Section 5.3.

To determine an optimal distribution of the vertical track stiffness at the transition is not trivial. The optimal vertical track stiffness depends on the operating conditions on the track, e.g. high-speed or freight [13]. A high stiffness value increases the dynamic wheel–rail contact forces, whereas a low stiffness value increases the energy dissipation, rail vibrations and noise. Further, the optimal distribution of the stiffness in a transition zone depends on what response that is evaluated. As an example, to minimise settlement, a vertical track stiffness that gives a low dynamic load on the foundation is required. On the other hand, if a vehicle response, e.g. ride comfort, shall be optimised, another distribution of the vertical track stiffness may be optimal. Hence, there is a trade-off between achieving minima of different track and vehicle responses. How to optimise a transition zone when several track and/or vehicle responses are taken into account simultaneously is further elaborated in Chapter 6.

## 5 Modelling of dynamic vehicle–track interaction

When comparing the railway mode to other transport modes, one of the unique features is the interaction between wheel and rail, which is the main source of vibrations when a train runs over a track. Since the birth of railways, researchers have been trying to model these vibrations accurately in order to understand how to reduce the noise and the deterioration of the vehicle and the track. The historical development of the modelling of the vehicle–track interaction is described briefly in Section 5.1.

The modelling of the track can be performed using analytical or numerical models and can be computed in either the time domain or the frequency domain [15]. The major benefit of analytical models is that they are computationally cheap, but they are, however, not well suited for including irregularities in vehicle, track and/or soil. Historically, analytical models have been used due to limited computational power, but nowadays numerical methods are commonly used. In Sections 5.2 and 5.3, a review of different techniques for the modelling of the dynamic vehicle–track interaction is given for slab track systems and transition zones, respectively.

If a model in the time domain or the frequency domain is the most suitable depends on the purpose of the simulation. Generally, analyses performed in the frequency domain are computationally cheaper than analyses in the time domain. However, frequency domain models are unable to handle non-linear characteristics. A problem with time domain models is that the material parameters cannot be frequency-dependent. In particular, the stiffness and damping of rail pads and under sleeper pads (USP) are known to be frequency-dependent.

### 5.1 Historical review

In this section, a brief review of the development of the modelling of dynamic vehicle–track interaction is given. Comprehensive reviews have been written by Knothe and Grassie [16] and Connolly et al. [15].

The first dynamic analysis of a track was carried out by Timoshenko in 1926 [17]. In this model, which handled the excitation by a harmonically varying stationary load, the track was modelled as a continuously supported Euler–Bernoulli beam. This model was further advanced by describing the rail as a Timoshenko beam, cf. [18], and by modelling the sleepers as rigid bogies at discrete locations, cf. [19]. Using Timoshenko beam theory provides a more realistic response at high frequencies since shear deformations are taken into account. In the beginning of the 1990s, the use of finite element models and analyses performed in the time domain started to increase. These methods were, however, computationally demanding (in particular with the computational power that was available at that time). By using a modal analysis for the track model, the computational



cost can be reduced. This was utilised by Lin and Tretheway [20] when solving the problem of a mass–spring–damper system moving on an elastic beam. By accounting for non-linear characteristics of the track and vehicle models in time domain analyses, various imperfections were analysed using coupled vehicle–track dynamics, cf. [21, 22, 23]. Today, finite element models analysing the dynamic vehicle–track interaction in the time domain are frequently used for a large variety of applications. In Sections 5.2 and 5.3, these kinds of models are reviewed for slab track systems and transition zones.

## 5.2 Slab track systems

When the dynamic vehicle–track interaction is modelled, Connolly et al. [15] describe that there are four cornerstones which are linked together: Accuracy, usability, parameter availability and computational cost. Depending on the purpose of the simulation, and what cornerstones that are considered most important, various simulation models can be used. Today, a common approach is to model the track using finite elements and the vehicle as a multi-body system. Zhai et al. [24] developed a three-dimensional model to investigate the overall vehicle–track system, both for ballasted track and slab track. The concrete slabs were modelled as elastic rectangular plates, and asymmetrical vertical irregularities on the two rails were considered. The model was further developed with a methodology to simulate transition zones between ballasted track, running on an embankment, and slab track, running on a bridge [25, 26]. Galvín et al. [27] modelled the vehicle as a multi-body system, the track with finite elements and the soil using a boundary element method. By using the coupled three-dimensional model, displacements and velocities were calculated for the car body, the track components and for the soil in the free field. Both simulation methods developed by Zhai et al. and Galvín et al. [26, 27] have been validated against physical tests.

Slab tracks have been modelled to analyse a variety of dynamic responses. For a floating slab track system, Li and Wu [28] investigated how the load transmission to the soil depends on the length of the slabs. Poveda et al. [29] analysed the fatigue life of slab track, and used the results to optimise the geometry of the slabs. Lei and Wang [30] modelled the track with finite elements and used a moving reference frame (the track model was assumed to be invariant along the track structure). In doing so, the vehicle acts at the same position on the rail throughout the simulation, and the study of the dynamic interaction between vehicle and continuous slab track can be solved at a lower computational cost. Zhu et al. [31] used a non-linear and fractional derivative viscoelastic (FDV) model of the rail pads to capture their complex characteristics. Coupled vehicle–track interaction simulations were performed and compared to simulations where the rail pad was described by a traditional Kelvin model. It was concluded that the FDV model captures the frequency-dependent dynamic stiffness and damping better than the Kelvin model. For discontinuous slabs, Zhang et al. [32] calculated the dynamic wheel–rail contact force due to a sinusoidal rail imperfection. In this work, linearised Hertzian theory was considered for the wheel–rail contact, and all simulations were performed in the frequency domain. Sadeghi et al. [33, 34] extended the model to three dimensions

and included non-linear properties of the wheel–rail contact. Yang et al. [35] used a so-called composite track model (the track model is divided into repetitive track elements that were combined to assemble the entire track model) and analysed how various track responses were influenced by rail irregularities.

In several papers, similarities and differences between slab track and ballasted track have been analysed. As examples, Blanco et al. [36] analysed and compared ballasted track with various traditional slab track designs (Rheda 2000, STEDEF and a floating-slab track), whereas Bezin et al. [7] compared ballasted track with two innovative slab track designs (one steel-concrete slab track and one embedded slab track).

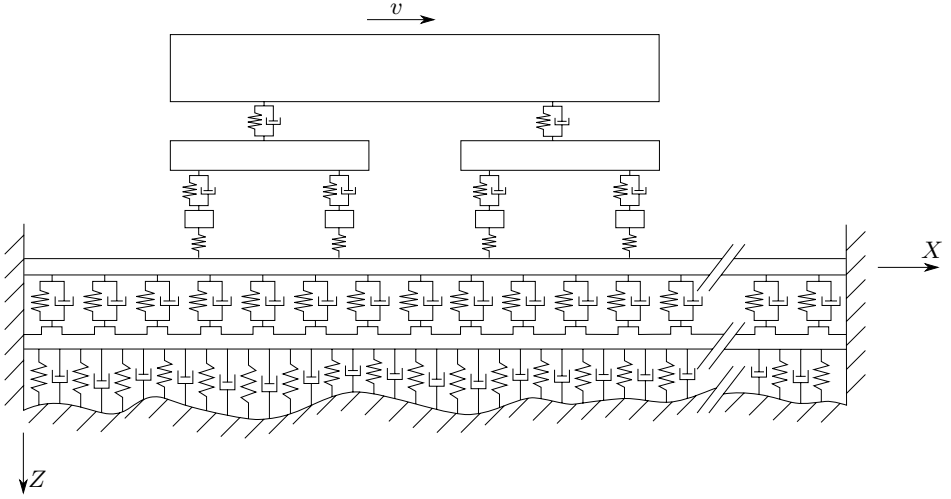


Figure 5.1: Sketch of track and vehicle model where the slab is modelled as one continuous layer of beam elements. The track model contains two layers of beams: rail and concrete panel. The concrete panel is supported by a Winkler foundation, where the prescribed (possibly random) variation in stiffness is indicated by the irregular ground surface. This slab track model is used to model the slab track type in Figure 2.5(a).

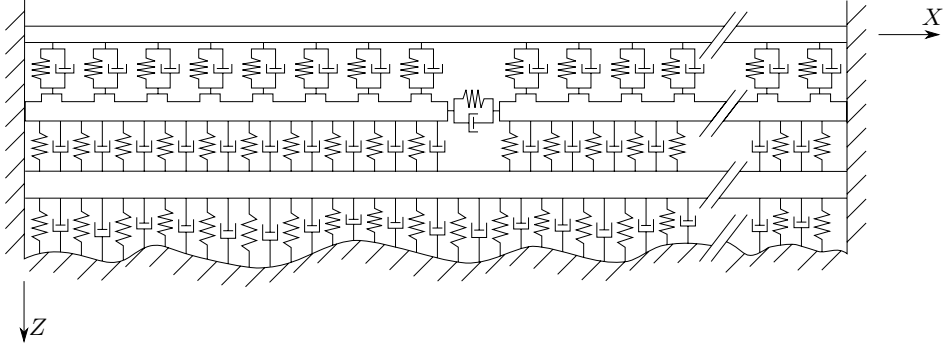


Figure 5.2: Sketch of track model where the slab is modelled as two layers of beam elements. The model contains three layers of beams: rail, discrete panels of concrete slab and continuous concrete roadbed. The concrete base is supported by a Winkler foundation. This slab track model is used to model the slab track type in Figure 2.5(b).

In **Paper A**, two types of slab track models are considered, see Figures 5.1 and 5.2. In Figure 5.1, the slab is modelled by one continuous layer of beam elements, and in Figure 5.2 the slab is modelled by two layers of beam elements. In the two-layer slab model, the upper layer containing the discrete slab panels is described by (coupled or decoupled) beams of a given length. The bottom beam layer (panel for the track model shown in Figure 5.1 and roadbed for the track model shown in Figure 5.2) is supported by non-interacting springs and dampers (Winkler foundation). The load from the vehicle is assumed to be symmetrically distributed between the two rails and, therefore, only half of the slab and one rail need to be considered. Moreover, only vibrations in the vertical direction are studied in this paper.

The slab track model in Figure 5.1 represents the type of slab track illustrated in Figure 2.5(a). In this model, it is assumed that the layers beneath the continuous concrete panel do not distribute any loads by bending. Hence, it is sufficient to model the rail and concrete panel as two layers of beam elements, while the remaining layers are incorporated in the Winkler foundation. Similarly, the slab track model in Figure 5.2 represents the slab track illustrated in Figure 2.5(b). Here, the reinforced roadbed may distribute the load by bending and, therefore, the rail, concrete panels and reinforced roadbed are all modelled as layers of beam elements, while the frost protection layer and soil are incorporated in the Winkler foundation. Note that the spring-damper coupling between the two concrete layers is continuously distributed.

These finite element models lead to large, coupled systems of equations of motion, which can be computationally intractable. The computational effort can, however, be reduced by using a modal reduction technique making the equations of motion decoupled. The equations of motion are then solved in the modal domain, and the solution is mapped to the spatial domain using the calculated eigenvectors. For more information about the track models and how the vehicle-track interaction problem is solved, see **Paper A**.

In **Paper B**, where a parametric study is conducted, the slab track model shown in Figure 5.2 is used. Here, the influences of a foundation stiffness gradient and track design parameters are investigated by calculating bending moments in the panels and loads between the roadbed and foundation.

### 5.3 Transition zones

Similar to the modelling of slab track systems, the modelling of transition zones can be achieved in various ways depending on the purpose of the simulation and the type of transition. As examples, effects of the variation in vertical track stiffness have been studied for transition zones of embankment–bridge type, cf. [12], embankment–culvert type, cf. [37], and slab track–ballasted track type, cf. [27, 14, 38].

The interest for the modelling of transition zones has grown in the last decade. One of the first analysis of transition zones was performed by Lei and Mao [39]. The influence of settlement and variations in the foundation stiffness and vehicle speed on the dynamic wheel–rail contact forces was investigated. It was concluded that a permanent settlement (modelled by changing the level of the rail) is the main source of the increased wheel–rail contact forces. However, the study only considered a ballasted track model and the transition zone was solely modelled by changing the rail level and/or foundation stiffness. For a transition zone between a ballasted track and a floating-slab track, Li and Wu [40] calculated rail displacements and wheel–rail contact forces when the influence of vehicle speed and the fundamental natural frequency of the floating-slab track was investigated. Galvín et al. used a coupled three-dimensional finite element model and boundary element model to simulate a transition zone between a ballasted track and a slab track [27]. The transition zone was divided into four sections of varying track structure to smoothen the changes in the vertical track stiffness, and the influence of the stiffnesses of the rail pads and foundation were investigated. Shahraki et al. [14] used simulations to calculate rail displacements, velocities and accelerations to examine the dynamic performance of a transition zone from a ballasted track to a slab track when using longer sleepers, auxiliary rails and/or improved subgrade. For a ballasted track, subjected to changes in soil stiffness, Lundqvist and Dahlberg constructed a single-objective optimisation problem where the dynamic component of the wheel–rail contact force was minimised with respect to the bed modulus of the soil in the vicinity of the transition [41]. The optimisation problem was solved using a Response Surface Methodology (RSM). Zakeri and Ghorbani [42] investigated how the displacements and accelerations of the rail can be reduced by gradually reducing the thickness of the slab in the transition zone.

In **Paper C**, transition zones are studied by using the simulation methodology presented in **Paper A**. The slab track model shown in Figure 5.1 is combined with a ballasted track model developed by Nielsen and Igeland [21], see Figure 5.3. By solving a multi-objective optimisation problem, the dynamic loads on the track are minimised with respect to selected track design parameters.

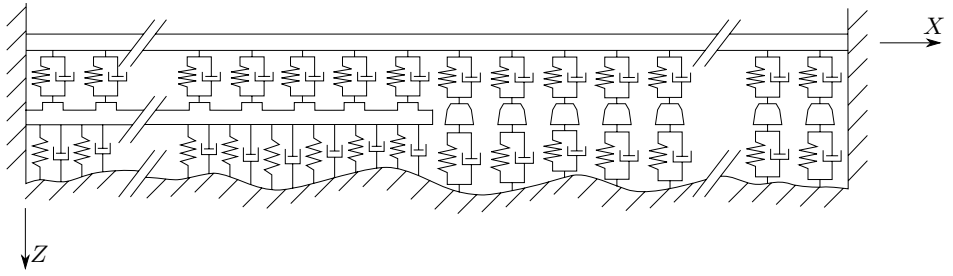


Figure 5.3: Sketch of complete track model with transition zone. The track model contains rail, sleepers and panel that are modelled as Rayleigh–Timoshenko beam elements. The sleepers and panel are supported by a Winkler foundation, where the prescribed (possibly random) variation in stiffness is indicated by the irregular ground surface.

## 6 Optimisation

When the optimal transition zone is designed in **Paper C**, a multi-objective optimisation problem is solved using a genetic algorithm called the Non-dominated Sorting Genetic Algorithm II (NSGA-II). In this chapter, genetic algorithms are briefly discussed in Section 6.1, while the NSGA-II is described in Section 6.2. Finally, some alternatives to genetic algorithms are discussed in Section 6.3.

### 6.1 Genetic algorithms

For the optimisation of several engineering problems, continuous methods and Response Surface Methodology (see Section 6.3) fail [43]. Typical reasons are several objective functions, computational cost and complexity of the objective function(s). A typical remedy is to use a heuristic method, where the use of Genetic Algorithms (GAs) is one of the most used methodologies. In GAs, the optimisation problem is solved by mimicking evolutionary phenomena in nature. All GAs have (similar to Darwinian natural selection) some kind of heredity, variation and selection. When a GA is applied, the members (also called chromosomes) in the first generation are usually selected randomly [43]. Each member is then assigned with a fitness value(s) based on the evaluation of objective function(s). The general idea is that a member with a higher fitness value has a higher probability of being part of the next generation. Producing the next generation is typically made by crossover (mixing two members from the previous generation) and mutation (adding more variation to the next generation).

### 6.2 Non-dominated Sorting Genetic Algorithm II

In **Paper C**, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) developed by Deb et al. [44] is employed. The algorithm can be described as follows: Initially, a parent population is generated randomly, and an offspring population is created based on binary tournament selection, recombination and mutation operators. Each member is assigned with a fitness value corresponding to its non-domination rank<sup>4</sup>. The best fronts are used to build up the next generation until all seats in the generation are occupied, which implies that elitism is ensured. Regarding the final front that is used to fill the last seats in the next generation, all members in the front will not fit, and a crowding distance control function is used to determine which members that shall be used. The crowding distance control function is defined to preserve diversity among the solutions. The generation of a new generation is shown schematically in Figure 6.1.

One iteration in NSGA-II has computational complexity  $\mathcal{O}(MN^2)$ , where  $M$  is the

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<sup>4</sup>The non-domination rank is determined by a non-dominated sorting approach that calculates non-dominated fronts.

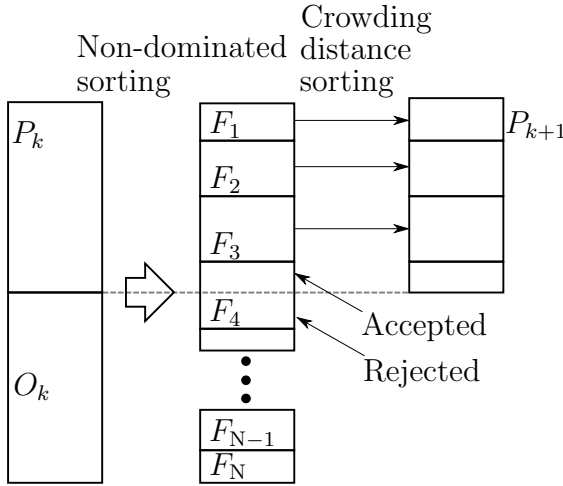


Figure 6.1: Scheme of how NSGA-II works.  $P_k$  and  $O_k$  denote the  $k^{\text{th}}$  parent and offspring generations, while  $F_1, F_2, \dots, F_N$  denote the non-dominated fronts. Based on Figure 2 in [44].

number of objective functions and  $N$  is the population size, which is significantly faster than its precursor NSGA, cf. [45]. In **Paper C**, the evaluation of the objective functions is the most computationally demanding step, which has to be performed for each member in each generation. The computational cost is, however, reduced by using parallel computations for each generation.

### 6.3 Alternatives to genetic algorithms

Classical continuous optimisation problems are typically solved with gradient-based algorithms, e.g. Newton's method (and modifications of it) or the steepest descent method [46]. The solution strategy for all gradient-based algorithms can be divided into several steps: initialisation, find descent direction, perform line search, update and termination check. Typically, the challenges with these types of algorithms are to determine the descent direction and step length. Depending on the used algorithm, the objective function is often required to be in  $\mathcal{C}^1$  (or even  $\mathcal{C}^2$ ). Other drawbacks with gradient-based algorithms are that only one objective function can be used per simulation and a minimum is not necessarily a global minimum if convexity of the objective function cannot be shown.

In optimisation problems, where the gradient cannot be calculated and expressed in closed form, the gradient can be calculated approximately using finite difference methods, e.g. forwards, backwards or central difference methods. An alternative approach when the gradient cannot be calculated is to use a so-called Response Surface Methodology

(RSM). In this method, the objective function is approximated with a meta-model, typically consisting of polynomials or splines (piecewise polynomial functions), and a regression model is used to fit the meta-model to the objective function. As soon as the coefficients to the meta-model have been determined, the objective function can be expressed approximately by the meta-model, and continuous optimisation techniques can be used. Since the objective function is smooth when the meta-model is employed, the optimal solution can be calculated and hence also an approximate solution to the original problem is obtained. In a post-processing stage, statistical methods can be used to estimate how good the fit of the meta-model is compared to the original objective function. As examples, Shevtsov et al. [47] and Nielsen and Fredö [48] used methodologies that are based on RSM in order to optimise railway wheels, while Lundqvist and Dahlberg [41] used RSM to optimise the dynamic component of the wheel–rail contact forces with respect to the foundation bed modulus.



## 7 Summary of appended papers

### **Paper A: Simulation of vertical dynamic vehicle–track interaction using a two-dimensional slab track model**

The vertical dynamic interaction between a railway vehicle and a slab track is simulated in the time domain using an extended state-space vector approach in combination with a complex-valued modal superposition technique for the linear, time-invariant and two-dimensional track model. Wheel–rail contact forces, bending moments in the concrete panel, and load distributions on the supporting foundation are evaluated. Two generic slab track models including one or two layers of concrete slabs are presented. Rail receptances for the two slab track models are compared with the receptance of a traditional ballasted track. The described procedure is demonstrated by two application examples involving: (i) the periodic response due to the rail seat passing frequency as influenced by the vehicle speed and a foundation stiffness gradient, and (ii) the transient response due to a discrete rail irregularity.

### **Paper B: Dynamic interaction between vehicle and slab track – Influence of track design parameters**

In this paper, the model developed in **Paper A** is applied to calculate the influences of foundation stiffness gradients and track design parameters on various track responses. In particular, the influences of the roadbed thickness on the load distribution on the foundation, and of the rail pad stiffness on the bending moment in the concrete panels, are investigated in two demonstration examples.

### **Paper C: Multi-objective optimisation of transition zones between slab track and ballasted track using a genetic algorithm**

In a transition zone between ballasted track and slab track, the vertical dynamic vehicle–track interaction is simulated in the time domain using the extended state-space vector approach described in **Paper A**. By considering a multi-objective optimisation problem solved by a genetic algorithm, the maximum dynamic loads on the track structure are minimised with respect to the selected design variables. From the solution of the optimisation problem, non-dominated fronts of the objective functions are obtained illustrating potential for a significant reduction of the dynamic loads. The influence of the length of the transition zone on the maximum dynamic loads is discussed. Since the transition zones are optimised neglecting the influence of wheel and rail irregularities, a methodology is proposed to evaluate the robustness of the optimal design by evaluating its performance when periodic rail irregularities with different combinations of wavelength and phase, relative to the position of the transition, are applied in the model.

## 8 Future work

The slab track model will be extended to three spatial dimensions, and it will be determined whether it is sufficient to model the panels as Rayleigh-Timoshenko beams or if a three-dimensional (3D) slab track model is necessary. Since the computational cost can be controlled by the number of used modes, the presented analysis procedure seems suitable also for a 3D model. Note, however, that more modes will be required in the 3D model if the same range of eigenfrequencies shall be used. Further, with a 3D model of the slab track including both rails, imperfections in the lateral direction, e.g. foundation stiffness gradients and an asymmetric loading on the track, can be analysed.

When considering the track models used in this thesis, only the ballasted track model has been validated against physical tests. In future work, also the slab track models need to be validated. The validation can be done using field tests and/or experiments in test rigs. An example of a full-scale test rig is described by Wang et al. [6]. From the test rig, displacements and accelerations of different track components have been measured for the CTRS (China Railway Track System) series using wheel drop tests.

Finally, when considering the Life Cycle Cost (LCC) of slab track vs ballasted track, the major benefits of slab track are less maintenance work and longer life. Research on fatigue analysis of slab track is, however, rather limited. In future work, the methodology presented in **Paper A** will be combined with an analysis of fatigue and crack initiation to investigate the life of slab track. As an alternative to classical deterministic approaches, stochastic simulations, cf. [49], can be used to determine the risk of failure.

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